

**Paper 4 (draft)**

**Soil Water Retention as a Function  
of Particle Size Distribution and Void Ratio**

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## Abstract

A continuous 4-parameter function, predicting soil water retention of poorly structured soils from particle size distribution and void ratio, was derived from Vereecken (1989) equation, which was adapted to particle size distribution with assumption of random particle arrangement. The function contains the Nimmo (1997)  $\beta$  parameter relating the pore opening to particle size and  $\kappa$  parameter, which is related to standard deviations of pore and particle size distributions. Two other empirical parameters determine the residual water content. The function predicted water retention of boreal forest soils with a higher accuracy than the original equation using conventional multiple linear regression. The poorer performance of linear regressions is attributed to prevailing nonlinear relationships and multicollinearity problems. The assumption of zero residual water content in physico-empirical models is incorrect particularly for finer soils because non-capillary retention of water flattens the curve in the low potential range. The new model fits the data better and reduces the error of prediction of soil water retention and may define the effect of aggregation on retention.

Soil texture is the dominant factor affecting soil hydraulic properties, particularly in the soils with poor structure. Of course, structural influence may be present even in a soil that has no macroscopic structural features (Nimmo 1997). However, it can be hypothesized, that in the soils, lacking contributions of organic matter, root and faunal activity, swelling-shrinking to structure formation this influence may still be determined by texture as sandy soils have nearly random arrangement of particles while clayey soils tend to have more organized microstructure.

Contents of sand, silt, and clay have been used extensively in the early attempts to predict soil water retention from basic properties. In most linear regression models predicting water content at fixed water potentials, sand and silt coefficients decrease while the clay coefficient does not change or shows a tendency to increase as water potential decreases (Gupta and Larson 1979, Rawls et al. 1982, Rawls et al. 1983, Puckett et al. 1985). The higher clay and silt content in the soil the higher water content at given water potential. The sand content is negatively related to water content in most models. Several authors got better estimates of soil hydraulic properties using sub-fractions, like fine sand (Puckett et al. 1985).

Obviously, the estimates could be improved by a parameterization of the entire particle size distribution curve. Similarity in the shapes of particle size distribution and water retention curves stimulated attempts to use same equation to fit both water retention and cumulative particle size distribution and then relate corresponding parameters using multiple linear regressions. The van Genuchten (1980) equation and modifications were used for this purpose in several reports (Havercamp and Parlange 1982, Rajkai et al.

1996, Schaap and Bouten 1996). This equation fitted well both the particle size distribution and water retention. The selection was also often based on its previous extensive use in hydrological modeling. Rajkai et al. (1996) found that one of parameters of the equation carries information on geometric mean pore/particle size while another parameter is more related to standard deviation of pore/particle size distribution. This observation provided a physical basis and further justification for the approach.

However, the relationships between pore and particle size distributions are far from simple and not necessarily linear (Nimmo 1997). A linear regression can only be a gross approximation to a non-linear relationship (Ratkowsky 1990). The explanatory variables are usually highly correlated, resulting in multicollinearity and biased estimations (Gunst and Mason 1980, Tabachnick and Fidell 1989). Besides, the multiple linear regression models are lacking potential for physical interpretation.

The physico-empirical approach (Arya and Paris 1981, Arya and Dierolf 1992) based on the assumption of random particle arrangement proved to be useful in model development. This assumption allows deriving water retention directly from particle size distribution data using simple arithmetic manipulations. The disadvantage of both models is that they are designed to produce discrete estimates of water content, not a continuous function, which reduces applicability of these models in practice of hydrological modeling. This also complicates estimation of the empiric parameters. The models with fixed parameter values may not always fit well to the real soil (Nimmo 1997). A similar approach was used in the (Haverkamp and Parlange 1982) model for sandy soils, but, apparently, this model could not be extended to heavier soils. The objective of this study is to obtain a continuous function predicting water retention from

particle size distribution, based on the assumption of random arrangement of particles and adjusted to real soils using nonlinear fitting.

## MODEL FORMULATION

Consider a desorption curve of an ideal porous medium formed by uniformly arranged multi-sized spherical particles, grouped by size as in Arya and Paris (1982), with zero contact angle between particle and water surfaces and no evaporation. For such medium, the slope of water retention curve  $\theta(R)$  and the slope of cumulative particle size distribution curve  $F(R)$  are related as

$$\frac{d\theta}{dR} = P \cdot \frac{dF}{dR} \quad (1)$$

where  $P$  is void volume to total volume ratio, which equals  $0.48$  and  $0.26 \text{ m}^3 \text{ m}^{-3}$  for cubic and tetrahedral arrangements, respectively. Integration of Eq.[1] gives an expression for water retention as a function of particle radius

$$\theta(R) = P \cdot F(R) + C \quad (2)$$

Constant  $C$  can be set arbitrary to  $\theta_r$ , the residual water content that marks the point where adsorption factors are getting involved in water retention (Jury et al. 1991). Then,

$$P = \theta_s - \theta_r \quad (3)$$

where  $\theta_s$  is saturated water content,  $\text{m}^3 \text{ m}^{-3}$ .

Substitution of [3] into [2] gives

$$\theta(R) = \theta_r + (\theta_s - \theta_r) \cdot F(R) \quad (4)$$

For a sandy soil, adsorption can be neglected and  $\theta_r$  assumed 0, which will bring Eq. [4] to the form derived by Havercamp and Parlange (1986). It was shown in the same paper that cumulative particle size distribution could be expressed using a modification of van Genuchten (1980) functional form. Rajkai et al. (1996) used the same equation in a simplified form:

$$F(R) = \frac{1}{1 + \left[ \frac{\alpha_t}{R} \right]^{n_t}} \quad (5)$$

where  $\alpha_t$  and  $n_t$  are empirical parameters.

Combination of [4] and [5] gives

$$\theta(R) = \theta_r + \frac{\theta_s - \theta_r}{1 + \left[ \frac{\alpha_t}{R} \right]^{n_t}} \quad (6)$$

A linear relationship can be assumed between particle radius and equivalent pore radius (pore neck in the case with desorption):

$$r = \gamma \cdot R \quad (7)$$

where  $\gamma$  is packing coefficient assumed constant for each arrangement of particles (Haverkamp and Parlange 1986). In fact,  $\gamma$  is not constant and the relationship [7] is not unique because not all the void space created by particles of same size is drained at same water potential; a small amount of water remains under menisci surrounding contact points between particles and is drained at lower potentials. For example, the remaining water contents were estimated 0.04 and 0.02  $\text{m}^3 \text{m}^{-3}$  for cubic and tetrahedral arrangements, respectively. The amount of meniscus water is related inversely to the squared water potential and directly to the cub of particle radius. This means a fast decrease in water content with a small decrease in water potential, which is especially true for smaller particles retaining most of the meniscus water. Therefore, the deviation can be considered negligible and the relationship between particle and pore opening radii still can be approximated by Eq. [7].

Knowing the relationship [7], water content can be expressed as a function of pore radius:

$$\theta(r) = \theta_r + \frac{\theta_s - \theta_r}{1 + \left[ \gamma \cdot \frac{\alpha_t}{r} \right]^{n_t}} \quad (8)$$

Pore radius  $r$  ( $\mu\text{m}$ ) can be converted into water potential using the equation of capillarity

$$r = -\frac{2 \cdot \sigma \cdot \cos \omega \cdot 10^{-6}}{\rho_w \cdot g \cdot \psi} = -\frac{\lambda}{\psi} \quad (9)$$

where  $\psi$  is soil water potential, kPa,  $\sigma$  is surface tension of water,  $\text{kg s}^{-1}$ ,  $\omega$  is contact angle between pore wall and water, degrees,  $g$  is acceleration due to gravity,  $\text{m s}^{-2}$ ,  $\lambda$  is a constant that equals approximately 149 at  $20^\circ\text{C}$  (Haverkamp and Parlange 1986).

Then, the soil water desorption function can be derived in Vereckeen (1989) form

$$\theta = \theta_r + \frac{\theta_s - \theta_r}{1 + \left[ -\frac{\gamma}{\lambda} \cdot \alpha_t \cdot \psi \right]^{n_t}} \quad (10)$$

with  $\alpha = \frac{\gamma}{\lambda} \cdot \alpha_t$  and  $n = n_t$

For a randomly arranged medium,

$$\gamma = \frac{\eta^{\frac{1}{3}}}{\beta} \quad (11)$$

where  $\eta$  is the void ratio and  $\beta$  is an empiric coefficient relating particle size to pore neck size (Nimmo 1997). Assuming a circular shape of pore neck section  $\beta$  can be calculated from particle radius and radius of the circle with an area equivalent to the pore neck section area: it comes to 1.9 and 3.2 for cubic and tetrahedral packing, respectively. The



range determined for random porous media experimentally is 2.1-2.6 (Nimmo 1997). It can be hypothesized that  $\beta$  values as high as 20, reported in the same paper for silt loams, could be because of consistent filling voids with smaller particles. Nimmo (1997) further adapted Eq. [11] to a non-uniform arrangement by direct incorporation of geometric standard deviation  $\sigma$  of size distribution

$$\gamma = \frac{\sigma \cdot \eta^{\frac{1}{3}}}{\beta} \quad (12)$$

assuming that pores tend to be wider in the more non-uniform medium.

It was also shown by Rajkai et al. (1996) that standard deviation of particle size distribution is inversely related to  $n_t$  parameter. Then, Eq. [12] can be rewritten with respect to particle size distribution

$$\gamma = \frac{\eta^{\frac{1}{3}}}{\beta \cdot n_t} \quad (13)$$

In non-swelling soils, the residual water content is essentially the amount of water absorbed to particle surfaces (Jury et al. 1991). Therefore, it is directly related to the specific surface area of soil, which, in turn, is inversely related to the particle radius. Assuming that geometric mean particle radius is directly related to  $\alpha_t$  parameter (Rajkai 1996) and multiplying the expression by bulk density to convert gravimetric water content to volumetric, the residual water content can be expressed as

$$\theta_r = \frac{1}{A+B \cdot \alpha_t} \quad (14)$$

where  $A$  and  $B$  are empirical parameters determined by both the geometric mean particle size- $\alpha_t$  parameter relationship and the capability of particle surface to adsorb water.

Saturated water content can be found from void ratio using

$$\theta_s = \frac{\eta}{1+\eta} \quad (15)$$

Combining Eqs. [10], [13], [14], and [15] and introducing  $\kappa$  parameter we obtain the model predicting water retention from particle size distribution and void ratio

$$\theta = \frac{1}{A+B \cdot \alpha_i} + \frac{\frac{\eta}{1+\eta} \cdot \frac{1}{A+B \cdot \alpha_i}}{1 + \left[ -\frac{\eta^{\frac{1}{3}}}{\lambda \cdot \beta \cdot n_i} \cdot \alpha_i \cdot \psi \right]^{\kappa \cdot n_i}} \quad (16)$$

Parameter  $\kappa$  is introduced to account for possible effects of deviation from random arrangement in a real soil on standard deviation of pore size distribution. In a random medium, values of  $\kappa$  should be close to 1.

## MATERIALS AND METHODS

A total of 14 sites were selected in mature conifer stands across west-central Alberta where a substantial amounts of forest operations occur in summer (Table 1). Most sites are located in the Southern Alberta Uplands ecodistricts of the Lower and Upper Boreal Cordilleran ecoregions (Strong 1992). The Lower ecoregion is dominated by lodgepole pine or white spruce with some aspen on well-drained soils while the Upper ecoregion is dominated by lodgepole pine with a small component of white or black spruce depending on the soil wetness (Corns and Annas 1986). Soils are predominantly Gray Luvisols that vary in degree of gleying and depth to mottles (Table 1).

At each site, samples were randomly collected from two locations in two plots approximately 10 by 40 m that had been protected from skidding traffic when the site was clearcut harvested. The two plots at each site were separated by a skid trail approximately 6 m wide. All samples were collected within a few days of harvesting. Undisturbed soil core samples, 3 cm in height and 5.2 cm in diameter, were collected in each sampling point from 5 and 10 cm depths using thin-walled brass rings (McNabb and Boersma 1993). These cores were sealed in plastic wrap and stored at +4°C to prevent fungal and bacterial growth and to maintain soil moisture at an initial level until analyzed.

Water retention was measured on these cores at pressures of -2, -5, -10, -30, -100, and -1500 kPa. Tempe pressure cells (Soil Moisture Equip. Co., Santa Barbara, USA) were used over the potential range from -2 to -30 kPa to reduce swelling (Reginato and van Bavel 1962). A 1500-kPa ceramic pressure plate extractor was used at lower

pressures. The pressure was continuously monitored using a pressure transducer connected to a CR7 Campbell Scientific datalogger that also operated an electrical solenoid valve maintaining gauge pressure within  $\pm 0.02$  kPa.

Bulk density was determined using the oven dry soil mass and volume of cores and particle density – using pycnometer method (Blake 1965 a, b). Void ratio was calculated from bulk density  $\rho$  ( $\text{m}^3 \text{m}^{-3}$ ) and particle density  $\rho_s$  ( $\text{m}^3 \text{m}^{-3}$ ) using

$$\eta = \frac{\rho_s - \rho}{\rho} \quad (17)$$

Particle size distribution was determined in bulk samples collected from each depth at each sampling point using combination of sieving and hydrometer methods (Day, 1965). Geometric mean particle diameter and standard deviation were calculated using the algorithm introduced by Shirazi and Boersma (1984).

Marquardt (1963) algorithm and SAS 6.11 software were used for nonlinear fitting. Initial values of parameters were evaluated with SigmaPlot 2.01 software. Eight models, commonly used to fit water retention curve and referred as to WRC models in Table 2, were fitted to every 6-point water retention data set obtained for every soil. Two PSD models commonly used to fit cumulative particle size distribution curve were fitted to every 14-point particle size distribution data set obtained for every soil. Two models were selected for further analysis, one for water retention and one for particle size distribution. Goodness-of-fit and simplicity of equation were used as criteria during selection. Simple models have better properties for nonlinear fitting (Ratkowsky 1989). Mean square error and square root mean square deviation were used as goodness-of-fit

measures; use of coefficient of determination was limited to linear models only (Kvalseth 1985).

$$MSE = \frac{\sum_{i=1}^n (\theta_m - \theta_e)^2}{n - p} \quad (18)$$

$$RMSD = \sqrt{\frac{\sum_{i=1}^n (\theta_m - \theta_e)^2}{n}} \quad (19)$$

where  $n$  is number of data points;  $p$  is number of parameters in the model;  $\theta_m$  ( $\text{m}^3 \text{m}^{-3}$ ) is measured water content;  $\theta_e$  ( $\text{m}^3 \text{m}^{-3}$ ) is estimated water content.

Distributions of dependent and independent variables were tested for normality; transformations were used as necessary. A standard multiple linear regression was performed for parameters of water retention function using two sets of independent variables. The first set included parameters of particle size distribution function and void ratio. The second set included the above variables with addition of bulk density, particle density, clay, silt, sand contents, geometric mean particle size and standard deviation of particle size distribution. Stepwise regression was used as an exploratory option to assist in selection of predictors. Significance level for entry and removal of variables was set to SAS 6.11 default value of 0.15. Multicollinearity and singularity diagnostics included examination of tolerance, variance inflation and condition index. A problem was anticipated if tolerance is lower than 0.01 or variance inflation and condition index are greater than 10 and 30, respectively (Tabachnick and Fidell 1989, Chatterjee and Price 1991, Freund and Littell 1986). The most common multicollinearity problem was

eliminated by separation of highly correlated variables. Linear regression was also used to verify relationships between parameters of particle size distribution function and geometric mean particle radius and standard deviation.

Eq. [16] was fitted to water retention of boreal forest soils twice with the SAS 6.11 PROC NLIN using Marquardt method and the number of iterations set to 200. Initial values were 0.4, 0.05, 1, and 2.2 for  $A$ ,  $B$ ,  $\beta$ , and  $\kappa$  parameters, respectively. Initial values for  $A$  and  $B$  parameters were obtained with SigmaPlot 2.01 software. The first fitting was performed on the entire data set to obtain parameter estimates for general equation. Then, the equation was fitted again to every 6-point water retention data set of every core separately. The variation in fitted parameters was analyzed for unaccounted relationships with predictors. The  $MSE$  and  $MRSD$  of fitting were compared with the results obtained on the same data set through estimation of parameters of water retention function using multiple linear regressions with parameters of particle size distribution and void ratio as predictors. Comparisons were also made with Arya and Paris (1981) and Arya and Dierolf (1992) physico-empirical models. Water content estimates were obtained in accordance with original procedures using recommended fixed values of 1.4 and 1 for  $\alpha$  and  $\alpha^*$  parameters and compared with water contents calculated using WRC7 equation fitted to every soil sample.

## RESULTS AND DISCUSSION

Soil bulk density, particle density, void ratio, sand, silt, clay contents, geometric mean particle diameter and standard deviation of particle size distribution are averaged by site and depth in Table 3. High bulk density values and the sharp increase in bulk density

with depth are typical of boreal forest soils (McNabb 1993). Decrease in particle density with depth is associated with a decrease in organic matter content. Soil textural class varies from sandy loam to clay loam but, at the majority of sites, soils are classified as silt loam or loam. Soil water contents measured at specific levels of water potential and averaged by site and depth are listed in Table 4.

Result of fitting of the original four-parameter van Genuchten (1980) equation (WRC6) was the best among eight water retention functions evaluated (Table 5). However, the WRC2 and WRC7 models were almost as good. The *MSE* and *RMSD* of two latter models are identical because they represent the same functional form. Taking into account good results of fitting and less complex structure, the WRC7 function was selected for further analysis. The WRC8 function did not converge on all cores and fitted poorly if converged, obviously, because the residual water content in this model was set to 0. The better fitting results of this model on the Swedish data set (Rajkai et al. 1996) could be explained by a possibility that the curve flattening point was missed somewhere in the large gap between water potentials 30 and 1500 kPa. Our data indicate that curve flattening starts at approximately 100 kPa.

Among the particle size distribution functions, the PSD1 modification gave the best fitting results (Table 5). However, the lesser complexity of PSD2 function and its similarity to already selected WRC7 function defined the choice and reasoning for previous selection of Eq. [5] during model development. Fitted parameters of selected water retention and particle size distribution functions are averaged by site and depth in Table 6.

Skewness of  $\alpha$ ,  $n$ , and  $\alpha_i$  distributions was eliminated by natural log transformation. The residual water content and  $\alpha$  parameter were predicted with reasonably high coefficients of determination using only two parameters of particle distribution and void ratio as independent variables (Table 7). No considerable improvement could be achieved by adding more independent variables because of increase in condition index beyond the acceptable limit. Coefficients of determination of  $n$  parameter are generally low (Rajkai et al. 1996, Schaap and Bouten 1996) that may be due to highly nonlinear relationships between standard deviation of water retention function and textural parameters in well-structured soils.

The correlation between geometric mean particle diameter  $\mu$  and  $\alpha_i$  parameter and between geometric standard deviation  $\sigma$  and inverse  $n_i$  parameter confirms the earlier assumptions that were used in formulation of Eqs. [13-14] (Fig.1). From the practical point of view, the regression equations combined with the Shirazi and Boersma (1984) algorithm can provide basis for  $\alpha_i$  and  $n_i$  calculation as a simplified alternative to nonlinear fitting.

The Eq. [16] fitted well to the entire data set (Table 8). Parameter estimates are close to the expected values and asymptotic errors are reasonably small. Asymptotic correlation between parameters is within acceptable limits (Bates and Watts 1988). The *MSE* and *RMSD* values of this equation greatly decreased due to incorporation of textural parameters and void ratio when compared with the results of fitting WRC7 function to the whole data set (Table 9). The improvement was not as great with the approach based on the estimation of parameters of this equation using multiple linear regressions. The consistent overestimation in the water potential range 1 to 20 kPa and



underestimation in the 20 to 500 kPa range is because of poorly defined  $n$  parameter (Table 7), which is responsible for the curve slope. Physico-empirical models can not be compared directly with other models by  $MSE$  and  $MRSD$  values because the residuals were calculated using estimates from fitted curve, not from measured data. However, the huge errors of both models are due to a severe underestimation of water contents in the medium and low potential range; the prediction is more adequate in the high range. An extreme case of such underestimation is shown in Fig. 2. The prediction based on capillarity alone without taking into account the adsorption factor is the main cause of underestimation in the low potential range. The assumption of cubic packing, led to overestimation of pore size and, consequently, to the curve shifting to the left in the medium potential range.

Fitting of Eq. [16] to the individual 6-point water retention data sets revealed a substantial variation in  $\beta$  and  $\kappa$  parameters among sites and depths. Both parameters show obvious dependence on  $\alpha_i$  parameter and geometric mean particle size  $\mu$  (Fig. 3). In coarser soils, where  $\alpha_i$  and  $\mu$  are higher than about 15 and 0.05 mm, respectively, these parameters have values close to those of a random medium. However, as  $\alpha_i$  and  $\mu$  decrease, the parameters show a tendency to decrease too. Logically,  $\beta$  parameter, as the ratio of particle and pore opening sizes, can not be smaller than 1.9 in a stable random medium. A decrease beyond this limit is likely caused by an aggregation of particles. The effect is more pronounced in soils with smaller geometric mean particle size, which reflects an increased amount of fine particles serving as a cement bonding larger particles into aggregates. Similarly,  $\kappa$  parameter is close to 1 in coarser soils indicating that standard deviations of particle size distribution and water retention function do not differ,

which could be expected for a random medium. A decrease in  $\alpha_i$  and  $\mu$  is associated with a decrease in  $\kappa$  parameter indicating an inadequate increase in standard deviation of water retention function. The obvious cause is a growing contribution of aggregation factor to pore size variability. The best-fit functions (Fig. 3) can be used to estimate the parameters from  $\alpha_i$  or  $\mu$ . Variation in  $A$  and  $B$  parameters could not be related to textural parameters or void ratio. It is likely driven more by variation in mineralogical composition of study soils, particularly by the amount of expanding minerals having large interlayer surfaces that adsorb additional amount of water. There is a potential to improve the model fit through a more accurate prediction of residual water content using additional information on soil mineralogy.

## SUMMARY

The Eq. [16] predicting soil water retention from parameters of particle size distribution was derived from Vereckeen (1989) equation adapted to particle size distribution with assumption of random particle arrangement. The function contains the Nimmo (1997)  $\beta$  parameter relating the pore opening to particle size. This parameter and the ratio of pore to particle size standard deviations adjust the model for a possible structural influence on water retention of boreal forest soils. The texture-defined component of the structural influence can be approximated by relationships of these parameters with texture.

The Eq. [16] predicted water retention of boreal forest soils with a higher accuracy than the original Vereckeen (1989) equation with parameters estimated using conventional multiple linear regression approach. It was associated with poorly

determined parameters of water retention function by linear regressions because of prevailing nonlinear relationships and multicollinearity problems. Assumption of zero residual water content in both physico-empirical models led to a severe underestimation of water contents in the low potential range because of non-capillary nature of factors flattening the curve in the low potential range.

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Table 1. Study site locations and soil types.

Site No.	Latitude	Longitude	Soil type
1	53o22'	117o00'	Orthic/Gleyed Gray Luvisol
2	54o32'	119o05'	Gleyed Gray Luvisol
3	53o57'	116o58'	Orthic/Gleyed Gray Luvisol
4	54o52'	115o15'	Orthic Gray Luvisol
5	53o00'	116o00'	Orthic Gray Luvisol
6	53o00'	116o00'	Orthic Gray Luvisol
7	52o10'	115o20'	Brunisolic Gray Luvisol
8	54o30'	119o00'	Orthic Gray Luvisol
9	54o00'	117o50'	Orthic/Gleyed Gray Luvisol
10	53o41'	117o50'	Orthic/Gleyed Gray Luvisol
11	51o45'	115o05'	Eluviated Distric Brunisol
12	54o54'	119o57'	Orthic Gray Luvisol
13	53o22'	117o00'	Gleyed Gray Luvisol
14	54o00'	117o50'	Orthic Gray Luvisol



Table 2. Functional forms commonly used to fit soil water retention curve (WRC) and cumulative particle size distribution (PSD).

Model	Equation	Parameters	Reference
WRC1	$\theta = \theta_r + (\theta_s - \theta_r) \cdot \left( \frac{\psi}{\psi_c} \right)^{\frac{1}{b}}$	$\theta_s, \theta_r, \psi_c, b$	Brooks and Corey (1964)
WRC2	$\theta = \theta_r + \frac{\theta_s - \theta_r}{1 + \left( \frac{\psi}{10^3 \cdot a} \right)^b}$	$\theta_s, \theta_r, a, b$	Bruetsaert (1966)
WRC3	$\theta = \left( \frac{a}{\psi} \right)^{\frac{1}{b}}$	$a, b$	Gardner (1970)
WRC4	$\theta = \theta_s \cdot \left( \frac{\psi}{a} \right)^{-b}$	$\theta_s, a, b$	Campbell (1974)
WRC5	$\theta = \theta_s \cdot \left\{ 1 + \ln \left[ \frac{1}{\left( 1 + \frac{\psi}{a} \right)^b} \right] \right\}$	$\theta_s, a, b$	Simmons (1979)
WRC6	$\theta = \theta_r + \frac{\theta_s - \theta_r}{\left[ 1 + (\alpha \cdot \psi)^n \right]^{\frac{1}{n}}}$	$\theta_s, \theta_r, \alpha, n$	van Genuchten (1980)
WRC7	$\theta = \theta_r + \frac{\theta_s - \theta_r}{1 + (\alpha \cdot \psi)^n}$	$\theta_s, \theta_r, \alpha, n$	Vereckeen (1989)
WRC8	$\theta = \frac{\theta_s}{1 + (\alpha \cdot \psi)^n}$	$\theta_s, \alpha, n$	Rajkai et al. (1996)
PSD1	$F(R) = \frac{1}{\left[ 1 + \left( \frac{\alpha_r}{R} \right)^{n_r} \right]^{\frac{1}{n_r}}}$	$\alpha_r, n_r$	Haverkamp and Parlange (1986)
PSD2	$F(R) = \frac{1}{1 + \left( \frac{\alpha_r}{R} \right)^{n_r}}$	$\alpha_r, n_r$	Rajkai et al. (1996)

Table 3. Soil bulk density, particle density, void ratio and textural parameters averaged by site and depth (n=4).

Site	Depth	$\rho^{\dagger}$	$\rho_s^{\ddagger}$	$\eta^{\S}$	Clay	Silt	Sand	$\mu^{\P}$	$\sigma^{\#}$	Texture
	cm	Mg m <sup>-3</sup>			%			mm		
1	5	1.15	2.52	0.80	13.0	44.0	43.0	0.08	11.23	Loam
	10	1.32	2.59	0.82	16.4	43.6	40.0	0.07	12.22	Loam
2	5	1.37	2.57	0.69	18.0	66.5	15.5	0.03	7.39	Silt loam
	10	1.44	2.62	0.83	22.9	62.1	15.0	0.02	8.17	Silt loam
3	5	1.28	2.57	1.07	12.0	54.0	34.0	0.06	9.63	Silt loam
	10	1.38	2.59	1.12	12.1	47.9	40.0	0.08	10.51	Loam
4	5	1.11	2.50	0.87	23.5	39.5	37.0	0.05	14.46	Loam
	10	1.33	2.54	0.76	22.1	35.7	42.2	0.06	15.06	Loam
5	5	1.19	2.54	1.20	18.0	26.5	55.5	0.11	14.89	Sandy loam
	10	1.20	2.62	1.06	14.3	29.3	56.4	0.13	12.93	Sandy loam
6	5	1.14	2.50	1.06	15.0	43.5	41.5	0.07	11.87	Loam
	10	1.21	2.59	1.20	12.1	37.9	50.0	0.11	11.49	Loam
7	5	1.08	2.54	1.11	26.9	56.0	17.1	0.02	9.56	Silt loam
	10	1.42	2.59	1.02	22.9	57.1	20.0	0.03	9.62	Silt loam
8	5	1.31	2.51	0.77	24.0	51.5	24.5	0.03	11.21	Silt loam
	10	1.52	2.52	0.68	31.4	46.4	22.2	0.02	12.26	Clay loam
9	5	1.38	2.49	1.07	18.0	58.5	23.5	0.03	9.37	Silt loam
	10	1.41	2.59	1.34	15.0	50.7	34.3	0.06	10.72	Silt loam
10	5	1.07	2.38	1.23	24.0	51.0	25.0	0.03	11.36	Silt loam
	10	1.19	2.52	1.05	24.0	49.5	26.5	0.03	11.79	Loam
11	5	1.05	2.37	1.26	15.0	53.0	32.0	0.05	10.30	Silt loam
	10	1.31	2.49	0.92	20.0	45.0	35.0	0.05	12.67	Loam
12	5	1.06	2.48	0.87	25.9	50.1	24.0	0.03	11.58	Silt loam
	10	1.19	2.53	0.84	29.5	31.0	39.5	0.04	17.60	Clay loam
13	5	1.08	2.42	1.46	25.0	50.0	25.0	0.03	11.63	Silt loam
	10	1.37	2.66	1.12	29.5	48.5	22.0	0.02	11.77	Clay loam
14	5	0.94	2.46	2.24	25.0	58.0	17.0	0.02	9.16	Silt loam
	10	1.28	2.47	0.96	22.0	58.0	21.0	0.03	9.43	Silt loam

<sup>†</sup> Bulk density.

<sup>‡</sup> Particle density.

<sup>§</sup> Void ratio.

<sup>¶</sup> Geometric mean particle size (Shirazi and Boersma 1984).

<sup>#</sup> Standard deviation of particle size distribution (Shirazi and Boersma 1984).

Table 4. Soil water content at 6 levels of water potential by site (mean and standard error, n=4).

Site	Depth	Water potential					
		-2 kPa	-5 kPa	-10 kPa	-30 kPa	-100 kPa	-1500 kPa
	cm	-----m <sup>3</sup> m <sup>-3</sup> -----					
1	5	0.43±0.02	0.39±0.02	0.33±0.02	0.27±0.02	0.19±0.02	0.18±0.02
	10	0.43±0.04	0.40±0.04	0.33±0.03	0.27±0.02	0.20±0.02	0.16±0.02
2	5	0.42±0.01	0.40±0.01	0.39±0.01	0.38±0.01	0.32±0.01	0.28±0.01
	10	0.44±0.02	0.41±0.01	0.40±0.01	0.38±0.01	0.33±0.01	0.30±0.01
3	5	0.53±0.04	0.51±0.04	0.49±0.04	0.39±0.04	0.25±0.02	0.23±0.02
	10	0.54±0.04	0.51±0.03	0.48±0.03	0.38±0.03	0.24±0.03	0.20±0.03
4	5	0.43±0.02	0.38±0.01	0.34±0.01	0.29±0.01	0.24±0.01	0.21±0.01
	10	0.43±0.02	0.40±0.02	0.37±0.02	0.30±0.03	0.27±0.03	0.26±0.03
5	5	0.48±0.03	0.39±0.02	0.34±0.01	0.28±0.01	0.19±0.01	0.18±0.01
	10	0.44±0.03	0.36±0.02	0.31±0.02	0.26±0.01	0.18±0.01	0.16±0.01
6	5	0.49±0.03	0.44±0.03	0.39±0.03	0.33±0.03	0.28±0.04	0.26±0.04
	10	0.50±0.04	0.43±0.04	0.37±0.04	0.29±0.02	0.24±0.02	0.20±0.02
7	5	0.51±0.03	0.47±0.03	0.45±0.03	0.41±0.03	0.35±0.04	0.32±0.05
	10	0.53±0.01	0.49±0.02	0.47±0.02	0.43±0.02	0.31±0.02	0.28±0.02
8	5	0.39±0.03	0.35±0.02	0.33±0.02	0.30±0.02	0.25±0.02	0.21±0.02
	10	0.38±0.02	0.35±0.02	0.33±0.02	0.31±0.02	0.26±0.02	0.23±0.02
9	5	0.51±0.02	0.47±0.03	0.44±0.01	0.37±0.02	0.31±0.01	0.29±0.02
	10	0.53±0.05	0.49±0.05	0.44±0.05	0.37±0.02	0.33±0.02	0.25±0.04
10	5	0.52±0.01	0.48±0.02	0.42±0.02	0.36±0.03	0.29±0.02	0.23±0.04
	10	0.47±0.01	0.43±0.01	0.39±0.01	0.36±0.01	0.30±0.02	0.26±0.03
11	5	0.55±0.04	0.50±0.04	0.45±0.04	0.33±0.02	0.27±0.03	0.24±0.05
	10	0.45±0.02	0.40±0.02	0.35±0.03	0.31±0.05	0.22±0.02	0.19±0.03
12	5	0.45±0.00	0.42±0.01	0.39±0.01	0.43±0.01	0.31±0.03	0.28±0.02
	10	0.45±0.03	0.42±0.04	0.39±0.03	0.32±0.02	0.29±0.02	0.27±0.02
13	5	0.55±0.06	0.52±0.06	0.44±0.09	0.40±0.06	0.34±0.05	0.30±0.06
	10	0.45±0.03	0.37±0.03	0.38±0.01	0.34±0.03	0.24±0.04	0.21±0.05
14	5	0.62±0.04	0.53±0.04	0.50±0.04	0.41±0.04	0.39±0.07	0.34±0.09
	10	0.45±0.01	0.41±0.01	0.38±0.00	0.34±0.01	0.31±0.02	0.28±0.03

Table 5. Results of non-linear fitting of WRC and PSD models of Table 2 to water retention and particle size distribution of every sample (n=112).

Model	MSE			RMSD			Converged %
	Min.	Max.	Average	Min.	Max.	Average	
				-----m <sup>3</sup> m <sup>-3</sup> -----			
WRC1	0.0000	0.0083	0.0011	0.0009	0.0528	0.0164	100
WRC2	0.0000	0.0042	0.0003	0.0012	0.0456	0.0107	100
WRC3	0.0001	0.0061	0.0009	0.0058	0.0637	0.0216	100
WRC4	0.0001	0.0081	0.0013	0.0058	0.0637	0.0223	100
WRC5	0.0001	0.0083	0.0018	0.0089	0.0745	0.0309	100
WRC6	0.0000	0.0032	0.0003	0.0018	0.0398	0.0103	100
WRC7	0.0000	0.0042	0.0003	0.0012	0.0456	0.0107	100
WRC8	0.0003	0.0098	0.0029	0.0130	0.0808	0.0409	87
				-----kg kg <sup>-1</sup> -----			
PSD1	0.0001	0.0016	0.0006	0.0111	0.0374	0.0211	100
PSD2	0.0002	0.0053	0.0015	0.0139	0.0674	0.0337	100

Table 6. Parameters of water retention and particle size distribution functions averaged by site, n=4.

Site	Depth	WRC7				PSD2	
		$\theta_s$	$\theta_r$	$\alpha$	$n$	$\alpha_t$	$n_t$
	cm	-----m <sup>3</sup> m <sup>-3</sup> -----					
1	5	0.46	0.17	0.08	1.07	16.37	0.82
	10	0.48	0.16	0.07	0.90	15.19	0.81
2	5	0.41	0.27	0.02	1.20	5.40	1.08
	10	0.46	0.28	0.04	0.68	4.39	0.98
3	5	0.52	0.22	0.03	1.94	11.96	1.09
	10	0.54	0.19	0.03	1.49	15.23	1.14
4	5	0.53	0.20	0.15	0.71	9.36	0.67
	10	0.44	0.26	0.08	1.43	12.14	0.67
5	5	0.63	0.16	0.20	0.76	28.96	0.69
	10	0.63	0.14	0.26	0.67	26.53	0.74
6	5	0.55	0.25	0.12	0.93	14.44	0.78
	10	0.62	0.19	0.16	0.79	21.27	0.79
7	5	0.55	0.31	0.06	0.76	4.65	0.81
	10	0.52	0.27	0.03	1.41	5.85	0.88
8	5	0.50	0.18	0.14	0.47	5.52	0.74
	10	0.43	0.22	0.07	0.59	3.37	0.65
9	5	0.53	0.28	0.06	1.09	8.29	1.00
	10	0.69	0.22	0.12	0.50	10.97	0.96
10	5	0.60	0.22	0.08	0.70	5.99	0.77
	10	0.58	0.24	0.13	0.55	7.14	0.81
11	5	0.57	0.24	0.07	1.27	10.03	0.87
	10	0.52	0.17	0.08	0.76	9.57	0.69
12	5	0.49	0.27	0.09	0.82	5.46	0.76
	10	0.47	0.27	0.07	1.25	4.95	0.64
13	5	0.61	0.27	0.05	0.61	6.00	0.76
	10	0.48	0.19	0.05	0.72	4.58	0.70
14	5	0.69	0.33	0.36	0.65	4.72	0.86
	10	0.57	0.27	0.27	0.59	5.45	0.86

Table 7. Linear regression models predicting  $\theta_r$ ,  $\alpha$ , and  $n$  parameters.

Set of predictors	Model	$R^2$
$\alpha_i, n_i, \eta$	$\theta_r = 0.1474 - 0.0474 \ln(\alpha_i) + 0.1228 n_i + 0.0941 \eta$	0.64
	$\ln(\alpha) = -2.2299 + 0.3518 \ln(\alpha_i) - 2.8911 n_i + 1.0628 \eta$	0.51
	$\ln(n) = -1.2099 + 1.1424 \ln(\alpha_i) + 1.1457 n_i - 0.2895 \eta$	0.27

Table 8. Estimates, asymptotic errors, 0.05 confidence intervals, and correlation matrix for parameters of Eq. [16] obtained by fitting to the entire data set.

Parameter	Estimate	Asymptotic standard error	Asymptotic 0.05 confidence interval	
			Lower	Upper
$A$	3.845	0.154	3.541	4.150
$B$	0.053	0.013	0.028	0.079
$\beta$	1.994	0.163	1.671	2.317
$\kappa$	1.215	0.080	1.057	1.373
<u>Asymptotic correlation matrix</u>				
	$A$	$B$	$\beta$	
$B$	-0.680			
$\beta$	0.379	0.201		
$\kappa$	0.230	-0.202	-0.500	

Table 9. Mean square error and square root mean square deviation of  $\theta$  prediction.

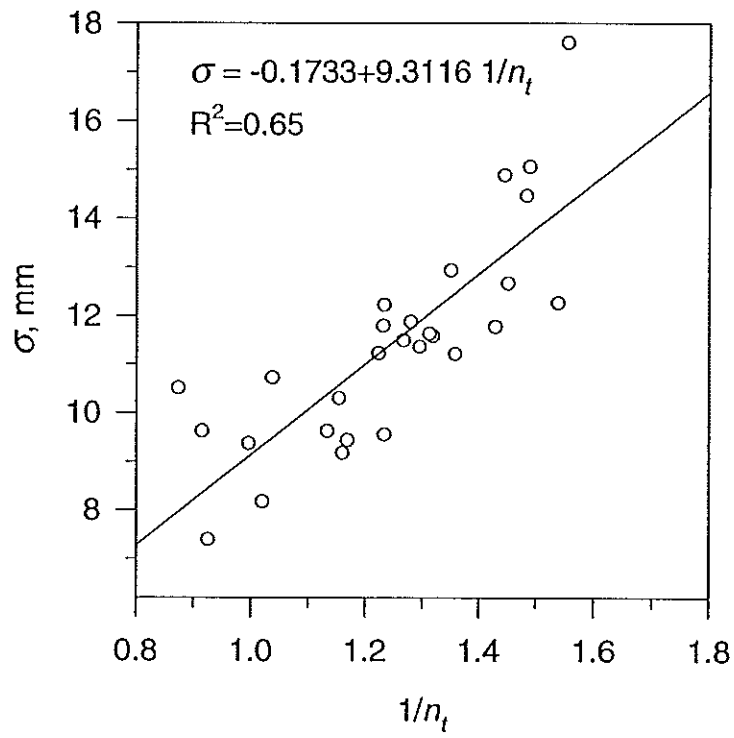
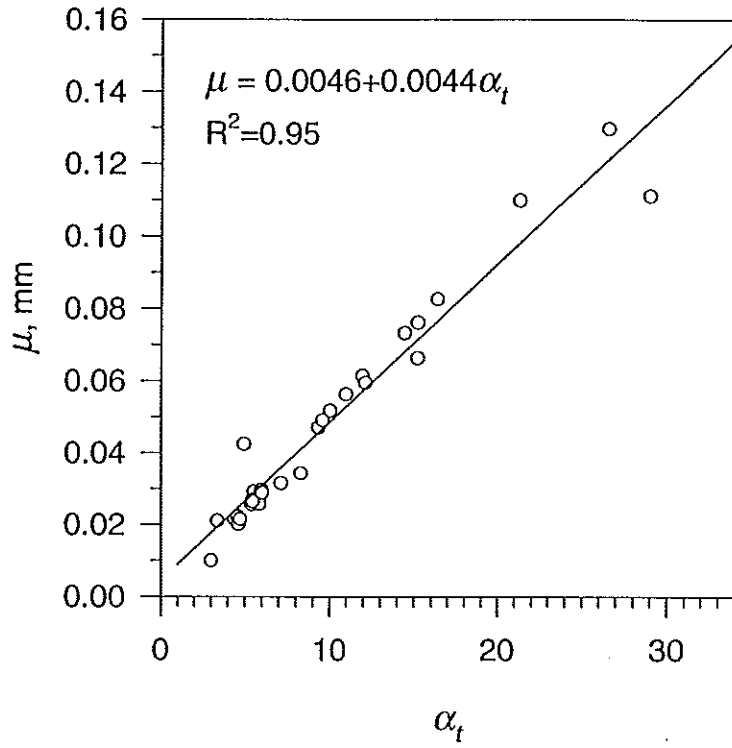
Model	$MSE$	$RMSD$ $m^3 m^{-3}$
WRC7	0.0028	0.0527
Non-linear fit Eq.[16]	0.0008	0.0288
WRC7 with parameters estimated by linear regressions (Table 6)	0.0026	0.0513
Arya and Paris (1982)	0.0325	0.1803
Arya and Dierolf (1992)	0.0262	0.1618

## List of Figures

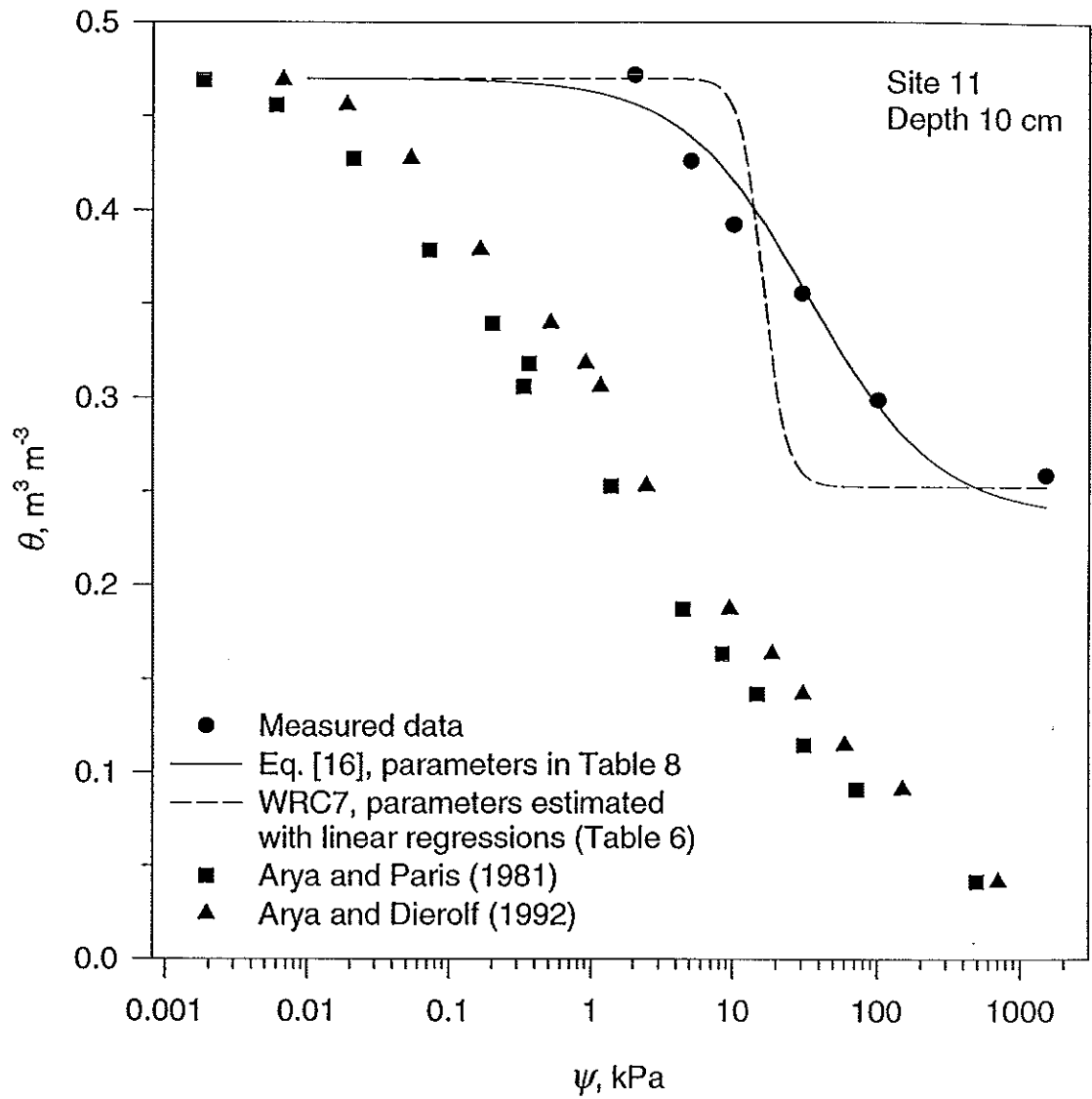
Figure 1. The relationships between  $\alpha_i$  and  $n_i$  parameters of particle size distribution function given by Eq. [5] and the geometric mean particle diameter  $\mu$  and standard deviation  $\sigma$  calculated according to Shirazi and Boersma (1984).

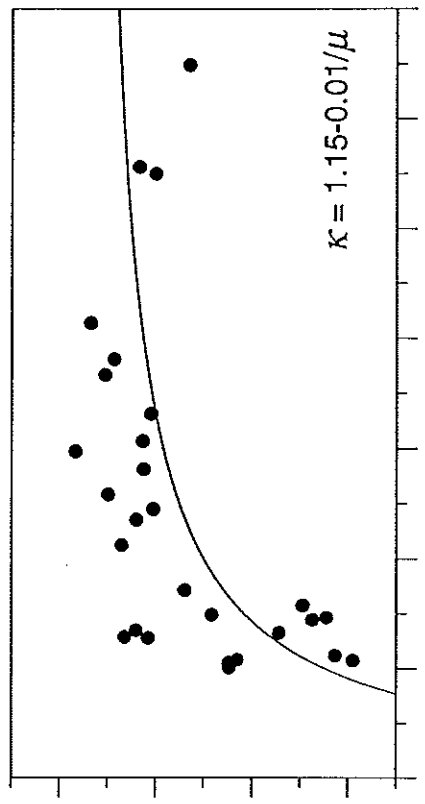
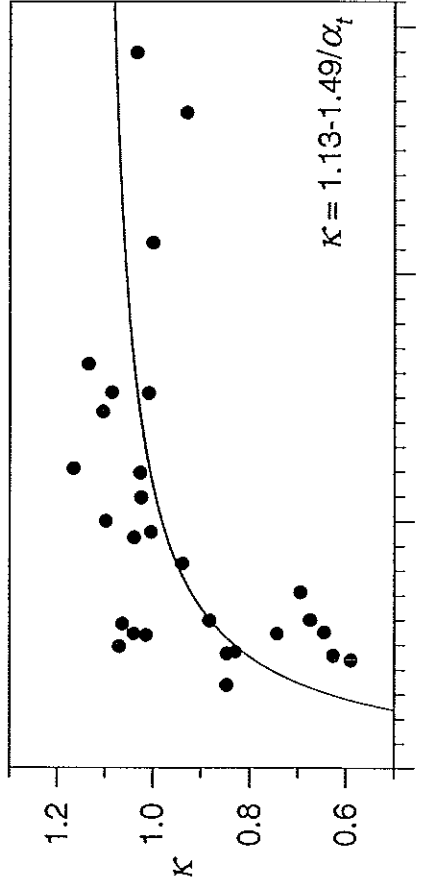
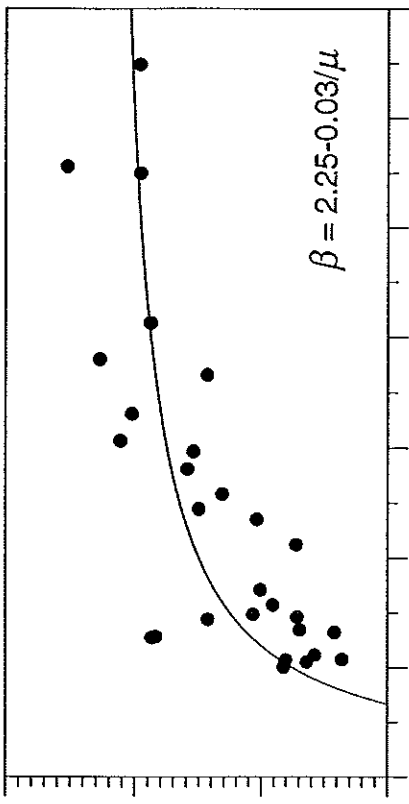
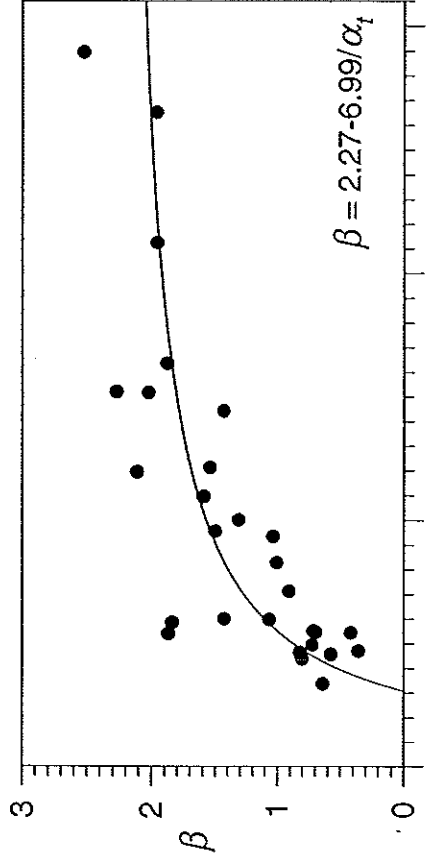
Figure 2. Parameters  $\beta$  and  $\kappa$  as a function of the  $\alpha_i$  parameter and geometric mean particle diameter  $\mu$ .

Figure 3. Prediction of water retention curve using Eq.16, Vereckeen (1989) model with parameters estimated by multiple linear regressions and physico-empirical models by Arya and Paris (1981) and Arya and Dielrolf (1992).









$\mu, \text{ mm}$

$\alpha_t$