FOREST GROWTH ORGANIZATION OF WESTERN CANADA

The Development of Stand Density Management Diagrams for Natural Lodgepole Pine and White Spruce Stands in Alberta

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FOREST GROWTH
ORGANIZATION OF WESTERN CANADA

Executive Summary

First approximation stand density management diagrams were developed for natural, even-aged lodgepole pine and white spruce stands in Alberta using the Provincial Growth and Yield Initiative plot database. A spreadsheet application was also developed to enable the visualization of stands, plot trajectories, growth model projections and the impact of thinning regimes and expected outcomes.

The project will help improve the management of the forest resource in Alberta through providing models and tools that enable forest managers to create crop plans that project the timing and scale of timber removal via commercial thinning operations.

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1. Overview

The Forest Growth Organization of Western Canada (FGrOW), an association of fRI Research has embarked on a research project supporting the implementation of the Enhanced Forest Management (EFM) practice of density management through Commercial Thinning (CT). One area of interest is the development of decision support systems that enable forest managers to create crop plans that project the timing and scale of timber removal via thinning operations. Stand Density Management Diagrams (SDMD) are essential tools in this exercise; however, they are currently not available for even-aged, pure pine and white spruce stands in Alberta, calibrated using local empirical data rather than growth models. Additionally, a practical tool for assessing decisions in the field when marking timber for removal to specific levels is also needed when laying out experiments or small demonstration areas.

This document describes the analysis that was undertaken to develop SDMDs for pure, even-aged lodgepole pine and white spruce stands using plot data submitted to the Provincial Growth and Yield Initiative (PGYI) database.

1.1 Introduction

Density management is the process of controlling the level of growing stock through initial spacing and subsequent thinning to achieve specific timber, vegetation, or wildlife management objectives (Long and Smith 1984).

SDMDs are average stand-level models that graphically illustrate the relationships among tree size, yield, density and density-dependent mortality at various stages of stand development (Newton and Weetman 1993). The development of SDMDs greatly facilitated the process of silviculture decision making to determine the timing and extent of thinning interventions.

The principal relationship that underlies SDMDs is the self-thinning rule that was originally described by Japanese scientists (Yoda *et al.* 1963) and later introduced into the North-American forestry literature by Drew & Flewelling (1979). The self-thinning rule defines the upper bound of the size-density relationship based on competition-induced mortality among stems in dense, even-aged stands. The general equation for the size-density relationship is given below:

$$\bar{v} = k_1 * N^{-3/2} \qquad \text{Equation 1.}$$

where $\bar{v}$ is the mean total stem volume, N is the number of live stems per unit area and k_1 is a species-specific constant.

In addition to mean stem volume, size may also be defined as plant weight, biomass, diameter[1] or height. The rule is also known as “-3/2 power law” because of the perceived relative stability of the exponent in

[1] One of the most familiar measures is the Stand Density Index (SDI) that describes the relationship between quadratic mean diameter (QMD) and trees per unit area in dense stands (Reineke 1933).

the relationship, although this has been debated by several researchers (Weller 1987, Bégin *et al*. 2001, Zeide 1987, Huiquan et al. 2000).

The self-thinning rule manifests itself as the maximum size-density line on a log-log scale:

$$\log(\bar{v}) = \alpha - \beta * \log(N) \quad \text{Equation 2.}$$

where α equals log(k_1) from Equation 1 and its value depends on the type of logarithm used, and β is the slope with a value of -3/2. Drew and Flewelling (1979) used the maximum size-density (self-thinning) line to introduce the concept of the relative density index (RDI) as the ratio of the observed actual density to the maximum density attainable in a stand with the same mean stem volume.

The self-thinning relationship has been shown to be independent of both stand age and site quality (Long and Smith 1984). This makes it ideal for density management applications providing the ability to compare different levels of growing stock and thus competitive stress, degree of site occupancy and growth potential, regardless of differences in site productivity and stand age. The stand density that is deemed ideal in the context of specific management objectives at rotation (e.g., target piece/log size, maximum growth, maximum yield, meeting wildlife habitat requirements etc.) can be projected forward or backward to a different stage of stand development (Long 1985) and thus help develop density management prescriptions.

1.2 Objectives

The main objectives of this project are to:

1. Develop first-approximation SDMDs for even-aged, pure lodgepole pine and white spruce stands in Alberta using local, representative plot data collected as part of the PGYI program.
2. Incorporate the size-density relationships into the Microsoft Excel based Commercial Thinning Tool that was developed for tree marking in experimental plot installations.
3. Develop additional tools in Microsoft Excel that enable the visualization of the new SDMDs (subject to available budget).

The project will help improve the management of the forest resource in Alberta through providing models and tools that enable forest managers to create crop plans that project the timing and scale of timber removal via commercial thinning operations.

2. Materials and Methods

2.1 Data

The data set used to develop the SDMDs in this study was obtained from the PGYI database in April 2022. The original data included 5,171 plots measured from 1949 to 2021 (18,731 measurements). The data was subjected to a thorough, two-phase screening process to ensure that only measurements from undisturbed, un-treated pure pine or pure white spruce stands were included.

Preliminary compilation of the entire data set was carried out in the official PGYI compiler (version March 14, 2022) using the SAS statistical software. Phase 1 of the data screening identified 1,612 plots (6,402 measurements) in the pure pine stratum and 410 plots (1,179 measurements) in the pure white spruce stratum[2].

The PGYI compiler only provided plot-level summaries by GYPSY species groups[3] therefore the tree-level data of the screened plots was re-compiled and re-summarized. All predicted heights for trees without a height measurement were retained as per the official PGYI compiler calculations. Any veteran trees were dropped, as well as any trees that were identified as being outside of the plot based on the tree measurement comments. Regeneration (trees below 1.3 m height) were excluded from the summaries.

Phase 2 of the data screening process included the identification and removal of suspect/erroneous measurements and any measurements with evidence of significant mortality that was unlikely to be competition-induced:

- Suspect plot measurements with large fluctuations in density attributable to significant changes in plot size and/or tagging limits over time.
- Plot measurements with significant drop in density due to a treatment (e.g., experimental plots in thinning treatments) based on treatment records and experiment documentation (Stewart *et al.* 2006). Control plots were retained where possible.
- Plot measurements with significant drop in density due to a catastrophic disturbance event (e.g., MPB attack, windthrow, fire etc.) based on disturbance records and plot measurement comments. Some of these plots were first visually identified with evidence of disturbance based on density and volume scatter grams. Measurements pre-disturbance were retained, whenever possible if they met all other criteria for inclusion.
- Plot measurements with a mean total tree volume of less than 0.01 m^3 were dropped as many of these measurements were in very young stands with large fluctuations due to the stochastic nature of stand development or erroneous measurements (for example, some plots did not include trees in the main plot at establishment – due to layout issues).

[2] At least 75% of the basal area of the plot is either Pl or Sw.

[3] PL group: pines and larches; SW group: spruces and firs; SB group: black spruce and AW group: all deciduous.

- For the purposes of SDMD development, only plot measurements with a live stem density greater than 100 stems per hectare were retained for analysis.

The final data set included 764 plots with 2,665 measurements for the pine, and 262 plots with 631 measurements for the spruce stratum. The developed SDMDs will be applicable in pure lodgepole pine and white spruce stands in Alberta where the respective conifer basal area is at least 75% of the total.

2.2 Methods

The general process for the development of SDMDs is outlined by Newton and Weetman (1993) as follows:

1. Determine the mean stem volume ($\bar{v}$) -density (N) relationship and RDI corresponding to
    a. The asymptotic $\bar{v}$-N relationship expressed as the maximum size-density line on a log-log scale (Equation 2)
    b. The lower limit of the zone of imminent competition mortality (ZICM), which is the stage of stand development where density-dependent mortality due to competition is likely to occur (Drew and Flewelling 1977).
    c. The approximate crown closure line.
2. Derive the quadratic mean diameter and top height[4] isolines.
3. Calculate merchantability[5] ratios as a function of mean stem volume.

2.2.1 Maximum Size-Density Relationship

The compiled data was used to assess if these plots could be used to estimate the maximum size density relationship in pure lodgepole pine and white spruce stands. The mean total tree volume was plotted against live tree density on a log scale for the pine and spruce strata (Figure 2-1).

Natural, fire-origin plot trajectories are identified in light grey and post-harvest regenerated (PHR) plots are shown in dark blue. Many stands are still in the development stage where there is no clear evidence of competition-induced mortality but there are stands at or near the self thinning stage where the volume-density trajectories follow the hypothetical line representing the maximum size-density relationship.

It is evident that there is not enough PHR stand data in the spruce stratum and that the data available in the pine stratum does not provide enough evidence regarding self-thinning (stands are still at an early stage in their development). Therefore, the natural fire-origin plot data was used to develop the self-thinning relationships.

Two maximum size density functions were developed using the methodology by Bokalo *et al*. (2007):

1. The mean maximum volume-density function and
2. The biological maximum volume-density function.

[4] The average height of the 100 fattest trees per hectare. Eligible trees include healthy, live, non-veteran trees without significant lean that have not been subjected to significant height growth impediment (broken or dead top).

[5] Merchantability was defined as the provincial utilization standard of 15 cm minimum stump diameter over bark, 10 cm top diameter inside bark, 30 cm stump height, 3.66 m usable length compiled to tree length utilization. This utilization limit will be using the notation 15/10/30 in this document.

The mean maximum volume-density function was fit by first placing the plots into 200 stems/ha density classes and ranking each observation within each density class based on the observed total stem volumes. Within each density class, plots that were above the 95th percentile were used for fitting the mean maximum volume-density function (Equation 2) using the base 10 logarithm. The slope of the line was determined based on the fitted data and not fixed at -3/2, but it was expected to be close.

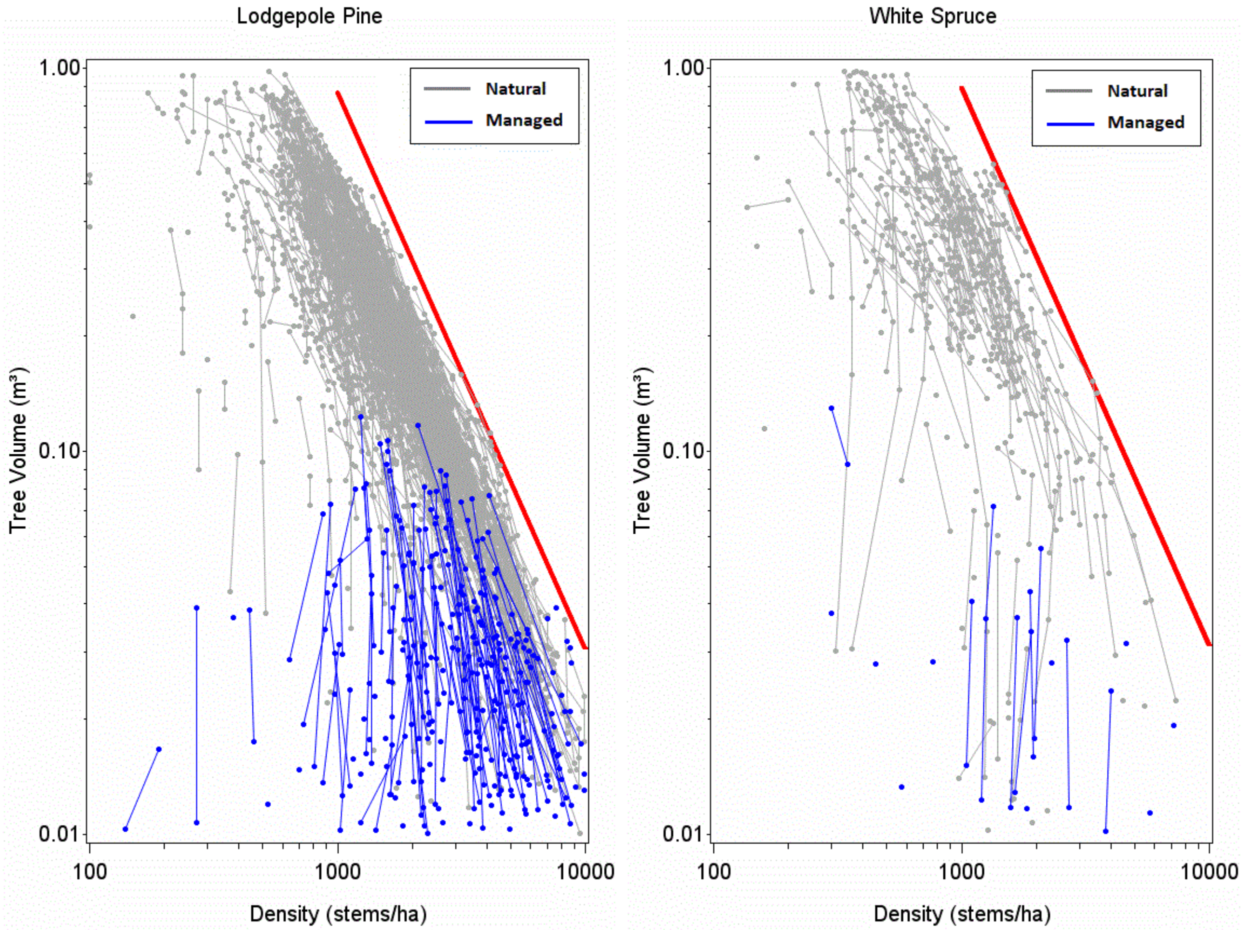

Figure 2-1. Volume-density trajectories for pine and spruce (red line indicates biological maximum)

The biological maximum volume-density line was positioned by simply adding the value of the largest positive residual to the intercept of the mean maximum volume-density function without changing the slope. This method shifts the line upward and forces all data points to be below the biological maximum line (Bokalo *et al*. 2007).

The mean maximum volume-density line is referred to the line corresponding to relative density index, RDI=1 (Drew and Flewelling 1979). Given that the line represents average stand conditions for self-thinning, it is possible that some stands may show an RDI greater than 1.

2.2.2 Lower Limit of the Zone of Imminent Competition Mortality

Drew and Flewelling (1979) defined the zone of imminent competition mortality as the stage of stand development where competition-induced mortality is likely to occur. Using this criterion, mortality data was derived from the PGYI plots and analyzed in reference to change in annual mortality rates by RDI. The range of annual mortality over 1% was used[6] as the threshold to identify the RDI where competition induced mortality becomes significant. In addition, the positive skewness of the diameter distribution of dead stems is also indicative of stands incurring density-dependent mortality due to competing for resources.

2.2.3 The Crown Closure Line

Before stands reach crown closure, trees are not competing with each other and their growth per unit area is simply proportional to the density. Once the crowns close, growth per unit area will increase with density; however growth per tree declines (Drew and Flewelling 1979). While there are crown measurements in the PGYI database, including crown radius and height to live crown, there was not enough time to explore the various methods employed in various studies (Newton and Weetman 1993, Long 1985, Drew and Flewelling 1979, Farnden 1996). Based on information in the available literature, an RDI=0.15 was chosen to represent the crown closure line until better information becomes available.

2.2.4 Quadratic Mean Diameter and Top Height Isolines

The relationship of quadratic mean diameter (QMD) to stem volume and density was expressed based on the following function (Sharma and Zhang 2007):

$$log_{10}(\bar{v}) = \alpha + \beta * log_{10}(QMD) + \gamma * log_{10}(N) \quad \text{Equation 3.}$$

where α, β and γ are coefficients estimated using linear regression analysis procedures on logarithmically transformed data.

Top height (THT) was incorporated using the function (Sharma and Zhang 2007):

$$\frac{1}{\bar{v}} = \alpha * THT^{\beta} + \gamma * THT^{\delta} * N \quad \text{Equation 4.}$$

where α, β, γ and δ are coefficients obtained using non-linear regression techniques.

It’s worth to note that Equation 4 is essentially a variation of the reciprocal equation of the competition-density effect (Kira *et al*. 1953) with the incorporation of THT:

$$\frac{1}{\bar{v}} = k_1 + k_2 * N \quad \text{Equation 5.}$$

where k_1 is a species-specific constant that depends on the stage of stand development and k_2 represents the intensity of the competition.

Isolines were obtained by rearranging the functions with respect to the mean stem volume and density by fixing QMD (Equation 3) or THT (Equation 4.) at specific values.

[6] Smaller scale mortality does occur due to biotic or abiotic factors before trees start competing. In addition, PSP data sets may also have minor measurement errors, missed trees, changing plot sizes etc. that contributes to net change in density counts. Using 1% as cutoff provides for eliminating some of these random factors.

2.2.5 Merchantability Ratio

Merchantability ratio (MR) was calculated as the ratio between merchantable volume per hectare at the provincial utilization of 15/10/30 versus the total gross biological volume per hectare for each plot measurement in the modelling data set.

The MR was related to the mean stem volume as follows:

$$MR = \frac{\alpha}{(1+\exp\left(\beta-\gamma*log_{10}(\bar{v})\right)} \qquad \text{Equation 6.}$$

where α, β and γ are coefficients obtained using non-linear regression techniques.

2.2.6 The Stand Density Management Diagram

The self-thinning line (mean maximum volume-density), the lower limit of the zone of imminent competition mortality, the approximate crown closure line, QMD and THT isolines were superimposed on a bivariate graph with total stem volume on the y-axis and stand density on the x-axis to develop the SDMD for even-age pure pine and spruce stands. An example of the pine SDMD is presented in Figure 2-2 with five measurements of a mature PSP (red dotted line) also shown. The blue lines represent the QMD isolines and the orange lines are the top height isolines.

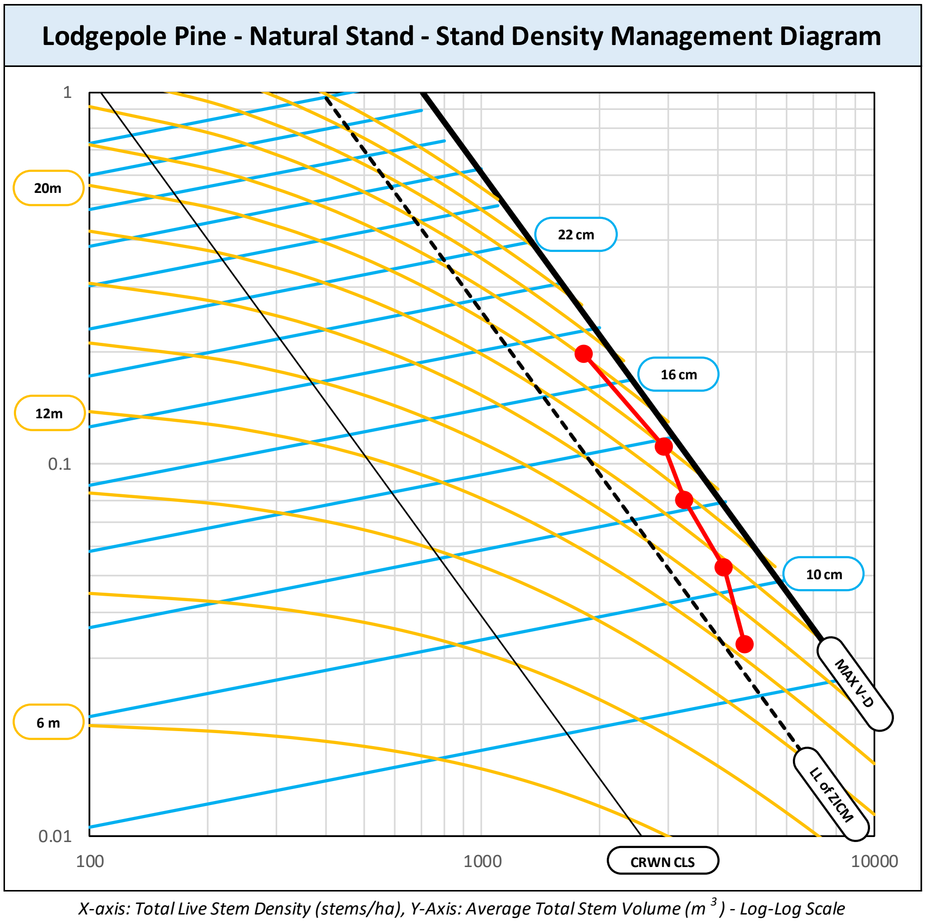

Figure 2-2. Stand Density Management Diagram for Even-Aged Lodgepole Pine Stands

3. Results

3.1 Data Summary

The summary statistics for the plot measurement data used in this study is provided in Table 3-1. All plot measurements were treated as equal for the purposes of the average calculations.

Table 3-1. Summary Statistics of the Plot Data by Stand Type

Stand Type	Variable	Mean	Std. Dev.	Minimum	Maximum
Lodgepole Pine (n=2,665)	Density (stems/ha)	2,030	1,615	100	13,400
	Basal Area (m^2/ha)	35.7	8.8	4.4	64.4
	Total Volume (m^3/ha)	298.9	110.4	16.0	642.5
	QMD (cm)	17.3	5.3	5.7	34.2
	Top Height (m)	19.9	4.1	6.2	30.0
	Stand Age (years)	89	30	33	234
White Spruce (n=631)	Density (stems/ha)	1,103	809	138	7,284
	Basal Area (m^2/ha)	36.2	11.7	3.4	71.9
	Total Volume (m^3/ha)	324.0	132.2	9.4	760.1
	QMD (cm)	22.9	6.4	5.3	45.8
	Top Height (m)	25.4	4.9	6.7	43.2
	Stand Age (years)	119	39	31	291

3.2 Fit Statistics

The fit statistics for equations 2, 3, 4 and 6 for lodgepole pine (PL) and spruce (SW) stands are presented in Table 3-2. All coefficients were significant at the 95% confidence level. The multiple coefficient of determination (R^2) was adjusted where applicable using the formula: (1-residual sum of squares/corrected sum of squares). All logarithms used in the equations are base 10.

Table 3-2. Fit Statistics by Stand Type and Equation

Stand Type	Equation	Coefficients				Fit Statistics		
		α	β	γ	δ	n	MSE	R^2
PL	Equation 2: log(v̄)=α-β*log(N)	4.1309	-1.4491			69	0.0108	0.97
	Equation 3: log(v̄)=α+β*log(QMD)+γ*log(N)	-5.1461	3.0522	0.2068		2,665	0.0026	0.98
	Equation 4: 1/v̄=α*THT^β+γ*THT^δ*N	8366.23	-2.8736	0.2978	-1.5902	2,665	12.517	0.90
	Equation 6: MR=α/((1+exp(β-γ*log(v̄))	0.9618	-4.6835	4.3917		2,665	0.0030	0.95
SW	Equation 2: log(v̄)=α-β*log(N)	4.1414	-1.4539			39	0.0181	0.92
	Equation 3: log(v̄)=α+β*log(QMD)+γ*log(N)	-4.9183	2.8919	0.1857		631	0.0037	0.97
	Equation 4: 1/v̄=α*THT^β+γ*THT^δ*N	241.97	-1.5991	18.9240	-2.8576	631	15.023	0.68
	Equation 6: MR=α/((1+exp(β-γ*log(v̄))	0.9782	-3.4476	2.3151		631	0.0018	0.82

As expected, the slope (β) in the mean maximum volume-density function was not significantly different from -3/2 at 95% confidence level for neither pine (p=0.1118) or spruce (p=0.3228).

The biological maximum volume-density lines used the largest positive residual for pine (0.1972) and spruce (0.1707) added to the intercept (α) of Equation 2 (Figure 3-3) as proposed by Bokalo *et al*. 2007.

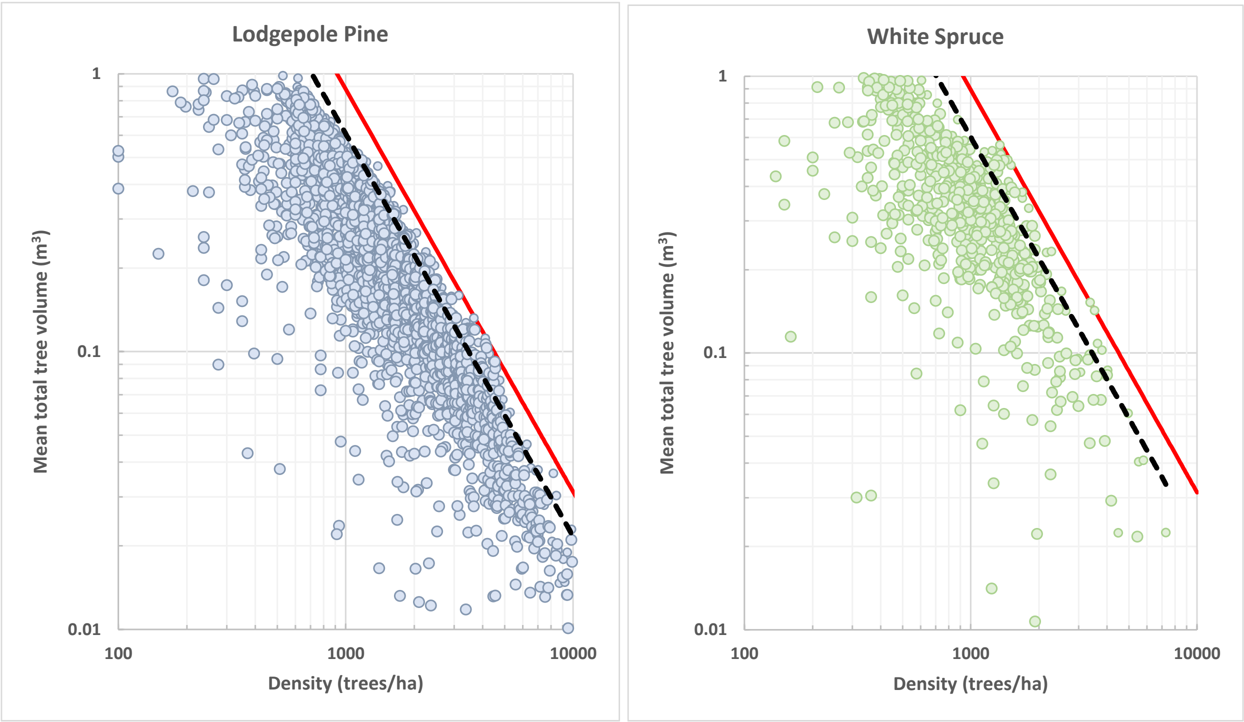

Figure 3-3. Biological (red solid) and mean maximum volume-density lines in pine and spruce stands

3.3 The Zone of Imminent Competition Mortality

To define the lower limit of the ZICM, mortality data was derived from the PGYI plots and analyzed in reference to change in annual mortality rates by RDI. The range of annual mortality over 1% was used as the threshold to identify the RDI where competition induced mortality becomes significant. The lower confidence limit of the annual mortality rate reached 1 % at RDI=0.55 in pine stands (Table 3-3) and at RDI=0.45 in white spruce stands (Table 3-4). Therefore, the lower limit of the ZICM was set to these values.

Table 3-3. Annual Mortality Rates by RDI in Pine Stands

RDI Class	Num. Obs.	Annual Mortality Rate Statistics			
		Lower Limit	Upper Limit	Mean	Std. Dev.
0.05	5	-0.7%	0.5%	-0.1%	0.5%
0.15	19	-0.1%	0.9%	0.4%	1.0%
0.25	60	0.4%	0.9%	0.6%	1.1%
0.35	127	0.8%	1.4%	1.1%	1.8%
0.45	254	0.8%	1.5%	1.1%	3.2%
0.55	432	**1.4%**	1.5%	1.4%	1.0%
0.65	519	1.5%	1.7%	1.6%	1.0%
0.75	312	1.8%	2.0%	1.9%	1.2%
0.85	138	1.9%	2.5%	2.2%	1.8%
0.95	35	1.8%	3.0%	2.4%	1.7%

Table 3-4. Annual Mortality Rates by RDI in Spruce Stands

RDI Class	Num. Obs.	Annual Mortality Rate Statistics			
		Lower Limit	Upper Limit	Mean	Std. Dev.
0.05	6	-3.1%	1.1%	-1.0%	2.0%
0.15	21	-3.5%	1.2%	-1.1%	5.1%
0.25	43	-2.6%	1.2%	-0.7%	6.3%
0.35	63	0.3%	1.5%	0.9%	2.2%
0.45	111	**1.1%**	1.6%	1.4%	1.5%
0.55	72	0.9%	1.6%	1.2%	1.5%
0.65	28	1.3%	2.1%	1.7%	0.9%
0.75	17	0.1%	2.2%	1.2%	2.1%
0.85	8	0.6%	1.8%	1.2%	0.7%

3.4 Validation of the QMD and Top Height Isolines

Because average total stem volume ($\bar{v}$) was expressed in terms of QMD and N in Equation 3 and as a function of THT and N in Equation 4, average prediction bias in estimating $\bar{v}$ was calculated using these equations for an independent data set of CFS plots that were not used in equation fitting. Most of these plots were treated (thinned/fertilized) in the 1940's to the 1970's as part of long-term silviculture trials in lodgepole pine stands in Alberta (Stewart et al. 2006).

There was a 7% over-prediction (bias=-0.009 m^3) of $\bar{v}$ using QMD and N in Equation 3 and a 7% under-prediction (bias=0.010 m^3) when using THT and N in Equation 4 that can be considered reasonable and within the ±10% benchmark bias that is generally used in Alberta for model validation (Table 3-5).

Table 3-5. Validation Statistics for Equations 3 and 4 in Lodgepole Pine Stands

Equation Type	Num. of Obs.	Variable ($\bar{v}$)	Mean	Std. Dev.	Lower 95% Conf. Limit	Upper 95% Conf. Limit	Minimum	Maximum
Eqn. 3 $\bar{v}$=f(QMD,N)	1,264	Actual	0.129	0.101	0.124	0.135	0.010	0.610
		Predicted	0.138	0.103	0.132	0.144	0.008	0.625
		Bias	*-0.009*	*0.016*	*-0.010*	*-0.008*	*-0.085*	*0.048*
Eqn. 4 $\bar{v}$=f(THT,N)	1,264	Actual	0.129	0.101	0.124	0.135	0.010	0.610
		Predicted	0.120	0.098	0.114	0.125	0.006	0.606
		Bias	*0.010*	*0.029*	*0.008*	*0.011*	*-0.124*	*0.106*

4. Discussion

First approximation SDMDs were developed for natural, even-aged lodgepole pine (Appendix I) and white spruce (Appendix II) stands in Alberta using the PGYI plot database. A spreadsheet application was also developed to enable the visualization of stands, plot trajectories, growth model projections and the impact of thinning regimes and expected outcomes (Figure 4-4).

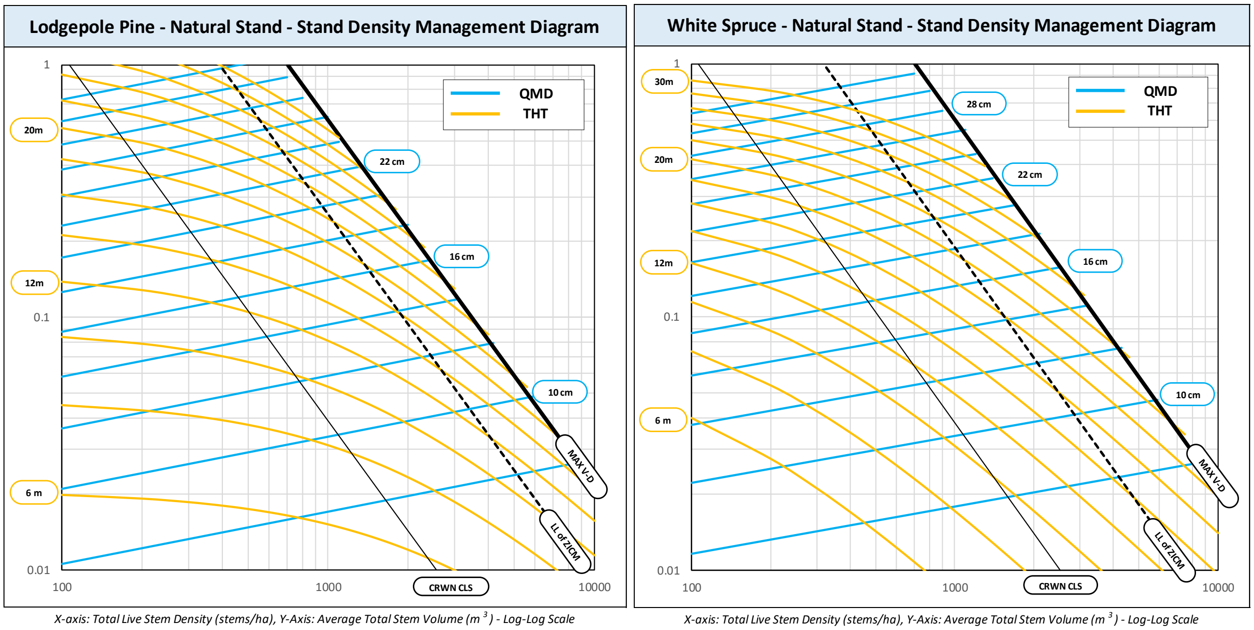

Figure 4-4. Stand Density Management Diagrams

These SDMDs graphically illustrate the relationships between yield, average tree size, density, QMD and top height at various stages of stand development. Although these relationships are expressed quantitatively, intra-specific resource competition and self-thinning concepts have been utilized in their derivation.

The temporal dependency of these processes governed by the intensity of competition and site quality as expressed by the relative density index and site index. Consequently, the diagrams can be used to describe the dynamics in natural lodgepole pine and white spruce stands using the top height isolines in combination with the appropriate site index curve[7] (Newton and Weetman 1994). Stand-level models such as GYPSY may be used to project anticipated growth responses after a thinning treatment using the residual densities and other stand attributes that are readily available from the diagrams.

Thinning scenarios can be superimposed on the diagram provided that the thinning treatment approximates the natural thinning process (e.g., randomly removing the smaller sized trees from spatially aggregated clumps). Spatial distribution changes after thinning and the timing of thinning and its impact

[7] The definition of top height in this study is compatible with the GYPSY site index equations (Huang et al. 2009) that is the official height-age model used in Alberta.

on growth response is not well understood. Further research is required before the full potential of these diagrams can be realized.

Enhancements of these diagrams could include more sophisticated statistical techniques regarding simultaneous fitting of a system of equations, the estimation of the self-thinning boundary line using stochastic frontier analysis or other enhanced statistical approaches and additional analysis regarding the crown closure line using live tree crown measurements from the PGYI database.

Future work may introduce the modelling of height and diameter distributions and associated potential of log merchandizing and the ability to project stands using the Mixedwood Growth Model (MGM) or other growth models that require tree size distribution details. Financial indicators such as harvest costs and net present value calculations may also be incorporated. Software development could help in presenting SDMDs that dynamically incorporate the anticipated changes after a thinning treatment (height-diameter relationship and growth response changes, extended rotation ages, changes to the lower limit of ZICM due to more even spatial distribution etc.). These analytical tools could provide forest managers with the ability to assess the consequences of various thinning scenarios in an interactive, visual format and help them develop a range of feasible strategies based on management objectives.

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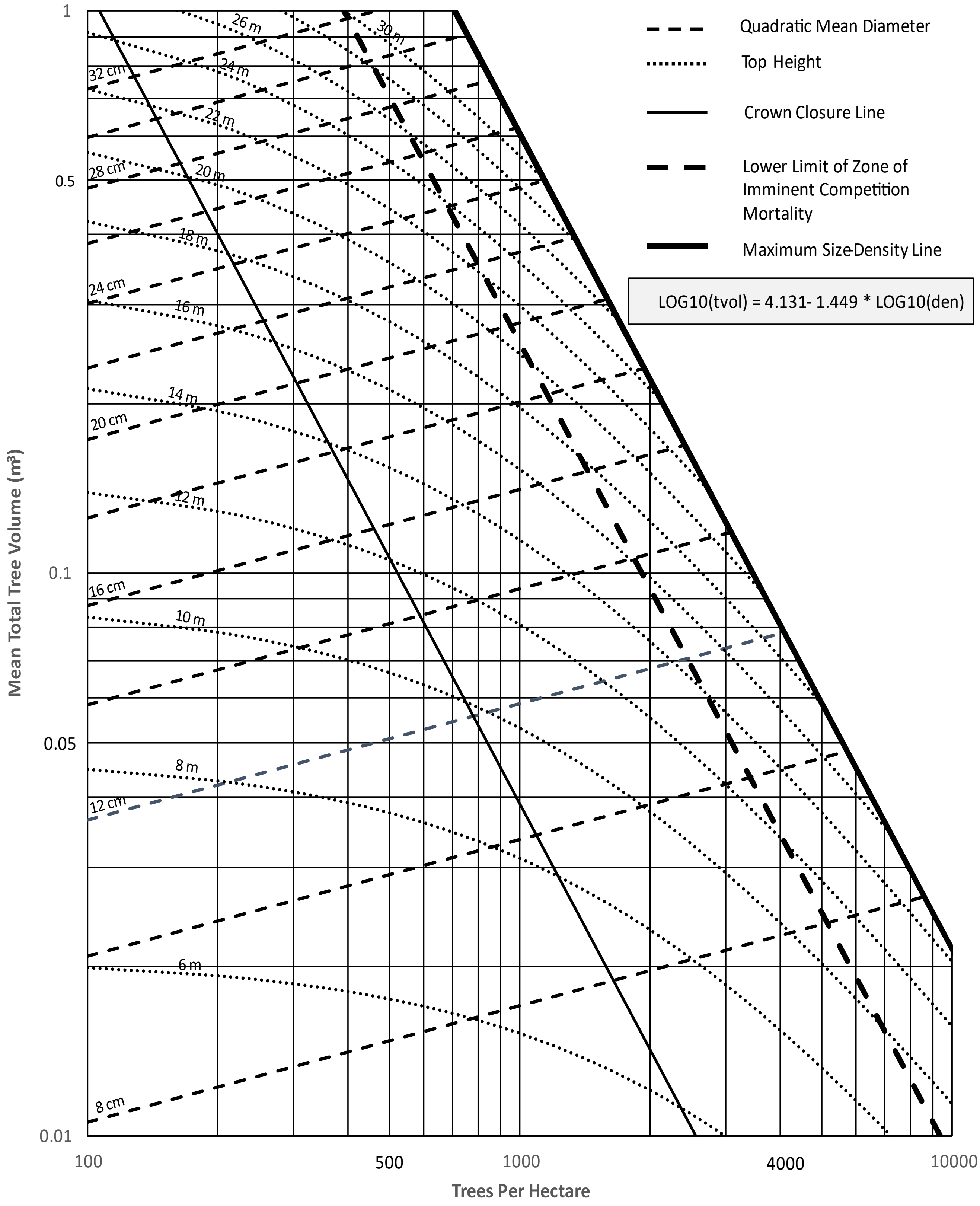
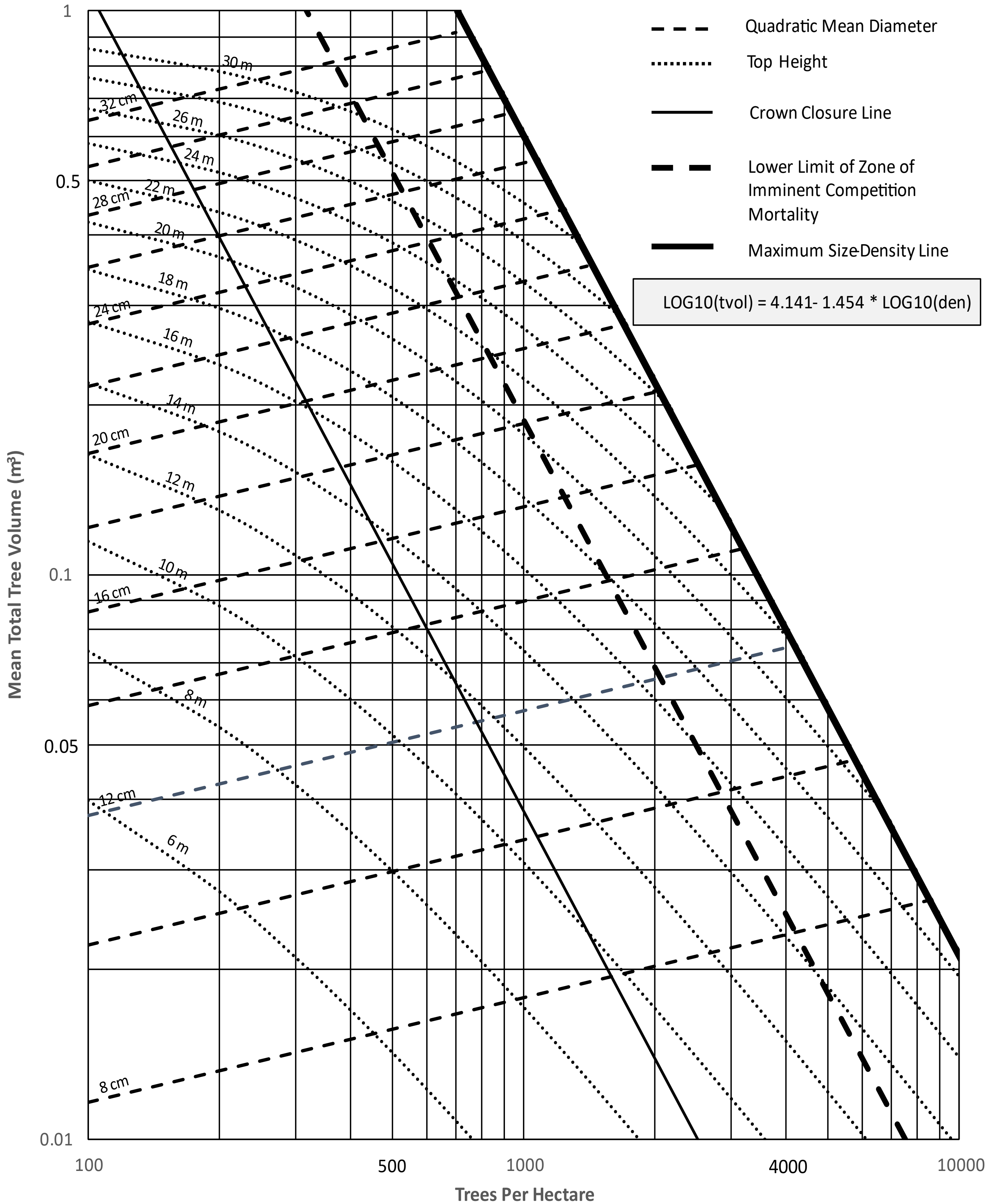
Appendix I – Natural Lodgepole Pine SDMD

NATURAL LODGEPOLE PINE STAND DENSITY MANAGEMENT DIAGRAM



Appendix II – Natural White Spruce SDMD

NATURAL WHITE SPRUCE STAND DENSITY MANAGEMENT DIAGRAM



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